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VOLUME TABLES AND EQUATIONS FOR OLD-GROWTH WESTERN HEMLOCK AND SITKA SPRUCE IN SOUTHEAST ALASKA

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by

James T. Bones, *Research Forester*
Institute of Northern Forestry

ABSTRACT

Field measurements collected in 1961 and 1962 from southeast Alaska logging operations were used for computing volume equations and constructing tree volume tables. Regression analysis was used for estimating three kinds of tree volume with three independent variables. Equations were produced in both weighted and unweighted forms in testing for unequal volume variances. Weighted equations and those containing form class as an independent variable were superior to unweighted equations and those not containing form class variables. Combined species equations were superior to single species equation forms.

These volume tables were constructed from 1,013 trees measured in 1961 and 1962 at 21 widely dispersed logging operations on the Tongass National Forest in southeast Alaska. The sample is representative of commercial timber stands now being harvested. The top and stump measurements used conform closely to present harvesting practices.[1/]

[1/] Bones, James T. Comparing inventory and woods utilization standards in southeast Alaska. Northern Forest Exp. Sta. Tech. Note No. 54, 7 pp., illus. 1962.

Merchantable top for board-feet equals 40 percent of d.b.h. (diameter at breast height), but not less than 6 inches inside bark; merchantable top for cubic feet equals 4 inches inside bark. Stump heights are those actually cut and average 18 inches. Seventy-four percent of the stumps were between 10 and 25 inches high.

Field measurements for the board-foot tables were d.b.h., d.i.b. (diameter inside bark) at 16-foot intervals between the stump and merchantable top, and merchantable height in number of 16-foot logs. Measurements for the cubic-foot tables were d.b.h., d.i.b. at varying lengths between the stump and a 4-inch top, and height in number of 16-foot logs to a top that equals 40 percent of d.b.h., but not less than 6 inches inside bark. Both 16- and 32-foot form class were computed for each tree. International 1/4-inch and Scribner[2/] log rule equations were used to compute board-foot tree volumes. Smalian's formula was used to compute cubic-foot volumes.

Equations were determined by regression analysis[3/] to predict volume for the two species by the three volume measures. Weighted and unweighted equations were solved to see which gave the best fit to the data. The weighted equation proved superior in that it was more successful in satisfying the assumptions underlying the regression analysis. The unweighted equation was:

$$V_k = b_0 + b_1D + b_2D^2 + b_3H + b_4D^2H + b_5 \frac{1}{D^2} + b_6F_j$$

where:

V_k = board-foot or cubic-foot tree volume

D = d.b.h.

H = merchantable height in 16-foot logs

F_j = 16- or 32-foot form class

b_i = constants (see table 2)

Observations having unequal variances of volume are normally weighted by the reciprocal or inverse of their variances. Similar results were obtained[4/] by weighting each variable of the equation by

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2/ Bruce, Donald. A formula for the Scribner rule. J. Forest 23: 432-433. 1925.

3/ Boles, J. N. 40 - series--stepwise regression system. (Unpublished report.) Calif. Agr. Exp. Sta., Dep. Agr. Econ., Univ. Calif., Berkeley. 43 pp. June 1962.

4/ Furnival, George M. An index for comparing equations used in constructing volume tables. Forest Sci. 7: 337-341. 1961.

$(D^2H)^{-2}$. This gave the following weighted equation:

$$\frac{V_k}{D^2H} = b_0' \frac{1}{D^2H} + b_1' \frac{1}{DH} + b_2' \frac{1}{H} + b_3' \frac{1}{D^2} + b_4' + b_5' \frac{1}{D^4H} + b_6' \frac{F_j}{D^2H}$$

where: b_i' = constants for the weighted equation.

All terms in the above equation must be multiplied by D^2H to convert it back to the unweighted form.

Table 1 presents standard errors of the estimated mean volumes in percent and multiple coefficients of determination (R^2), an index of the amount of variation accounted for by regression for the unweighted equations. Because the variation associated with D^2H is removed by the weighting technique before the regression is performed, R^2 could not be retrieved for the weighted form.

Weighting reduced standard errors of the equations for all species, form classes, and types of volume. The Sitka spruce equations are stronger than the western hemlock equations although based upon a smaller sample.

Form class improved the equation precision. Thirty-two-foot form class improved the Sitka spruce board-foot equations more than 16-foot form class; for western hemlock, the reverse is true. The cubic-foot equations for both tree species are better using the 16-foot form class.

Values for intercept constants and regression coefficients of the weighted equations are given in table 2. Equations containing these values can be used directly in electronic computer programs when processing timber cruise data.

In some cases, illogical volume estimates are obtained in the low diameter classes and outside the range of the original tree data. The equations should therefore be tested to assure that valid volume estimates are being given within the range of the tree data being processed.

Tables 3 to 8 will fill the needs of those workers desiring tree volume tables for field volume estimates or calculating volumes by hand at field stations. The form class equations were not used for these tables, because the increased precision by including the form class variable is overshadowed by the difficulty of accurately measuring form class in the field.

Although F-tests indicate that individual equations for spruce and hemlock are not significantly superior to a combined equation, separate volume tables are presented. The divergence of volumes for these species in the low and high diameter and height classes indicates an advantage of this separation.

Table 1.--*Precision of alternative board-foot and cubic-foot volume equations*

Species	Form class used	Standard error of estimate: Weighted equation	Standard error of estimate: Unweighted equation	R^2 for unweighted equation[1/]	Number of trees
		--------- Percent --------			
BOARD FEET (SCRIBNER RULE)					
Sitka spruce	None	8.57	18.31	0.972	266
	16-foot	7.41	13.92	.984	
	32-foot	7.16	12.76	.987	
Western hemlock	None	11.12	18.69	.961	747
	16-foot	8.20	14.29	.977	
	32-foot	8.81	14.27	.977	
Combined	None	10.55	18.95	.965	1,013
	16-foot	8.00	14.64	.979	
	32-foot	8.39	14.28	.980	
BOARD FEET (INTERNATIONAL 1/4-INCH RULE)					
Sitka spruce	None	8.90	17.90	.972	266
	16-foot	7.20	13.37	.984	
	32-foot	6.96	12.24	.987	
Western hemlock	None	10.78	17.97	.961	747
	16-foot	7.91	13.70	.978	
	32-foot	8.54	13.68	.978	
Combined	None	10.26	18.27	.965	1,013
	16-foot	7.76	14.03	.979	
	32-foot	8.15	13.68	.980	
CUBIC FEET					
Sitka spruce	None	9.80	14.32	.978	266
	16-foot	8.93	12.82	.982	
	32-foot	9.10	12.01	.985	
Western hemlock	None	10.49	16.55	.961	747
	16-foot	9.21	14.33	.970	
	32-foot	9.37	14.60	.969	
Combined	None	10.37	16.40	.966	1,013
	16-foot	9.26	14.56	.973	
	32-foot	9.40	14.38	.974	

[1/] The multiple coefficient of determination (R^2) could not be estimated for the weighted form of the equations.

Table 2.--*Constants for weighted volume equations of southeast Alaska tree species*

Species	Form class used	Intercept constant	Independent variable					
			D	D^2	H	D^2H	$1/D^2$	F_j
CONSTANTS FOR SCRIBNER BOARD-FOOT EQUATIONS								
Sitka spruce	None	--	--	-0.22256	-21.06369	0.34399	7,219.0713	--
	16-foot	-304.2926	--	- .54625	-59.0693	.39191	-14,084.00	7.25558
	32-foot	-187.38916	--	- .53915	-86.65427	.39565	-14,794.00	8.23733
Western hemlock	None	--	--	--	- 8.6308	.28348	--	--
	16-foot	484.10886	--	- .11060	-29.73546	.31567	--	6.97891
	32-foot	84.10443	--	- .39524	-64.80464	.36082	-13,497.00	5.45906
Combined	None	--	--	- .11546	-14.09611	.31378	3,653.44820	--
	16-foot	-464.35522	--	- .15525	-33.34669	.32678	--	6.88730
	32-foot	- 97.28730	--	- .43216	-68.45546	.37037	-13,247.00	5.81713
CONSTANTS FOR INTERNATIONAL 1/4-INCH BOARD-FOOT EQUATIONS								
Sitka spruce	None	--	--	- .22276	-13.51970	.35417	7,011.9961	--
	16-foot	-315.64641	--	- .56546	-53.46286	.40490	-15,422.00	7.57891
	32-foot	-193.47406	--	- .55793	-82.25466	.40879	-16,157.00	12.98374
Western hemlock	None	- 9.6512439	2.6510883	- .17949617	- 7.4021307	.320969	1,876.4131	--
	16-foot	-766.44019	17.729338	- .43793342	-23.419670	.32849136	12,662.188	7.3746037
	32-foot	- 96.860265	.68289865	- .42034489	-59.569917	.37298366	-14,107.924	5.707146
Combined	None	--	--	- .11259	- 6.38331	.32316	3,361.3556	--
	16-foot	-526.97453	5.94988	- .35391	-31.76929	.35106	--	7.22788
	32-foot	435.58416	- 6.34553	- .34539	-63.97241	.38436	-19,678.00	6.04978
CONSTANTS FOR CUBIC-FOOT EQUATIONS								
Sitka spruce	None	16.30566	--	--	- 3.09606	.04983	--	--
	16-foot	70.22160	--	- .07640	- 9.83336	.06079	- 4,421.30140	.87777
	32-foot	67.40064	--	- .04095	-10.97363	.05611	- 3,067.87990	.93612
Hemlock	None	--	--	.04915	--	.03912	269.92685	--
	16-foot	- 59.64022	--	.04926	- 1.68514	.03965	--	.83469
	32-foot	- 40.37296	1.53311	--	- 5.27334	.04321	--	.66296
Combined	None	--	--	.04589	--	.04015	391.15799	--
	16-foot	- 61.58925	1.13797	--	- 3.60359	.04505	--	.82038
	32-foot	- 8.30136	--	--	- 7.10379	.04834	- 1,853.8527	.70015

Table 3.--*Volume table for old-growth Sitka spruce in southeast Alaska*[1/]

(In board feet, Scribner rule)

D.b.h. (D)[2/] (inches)	Merchantable height in 16-foot logs (H)[3/]										Basis[4/]
	1	2	3	4	5	6	7	8	9	10	
12	47	75	103	132	160	189					5
14		86	132	179	225	271					15
16		105	172	239	306	373					22
18		131	221	312	402	492					24
20		162	279	395	512	628					26
22			343	489	634	780	925	1,071			28
24			416	593	770	947	1,124	1,301			19
26			495	706	918	1,129	1,340	1,552			18
28			581	829	1,078	1,326	1,575	1,824			21
30			673	962	1,250	1,539	1,827	2,116			20
32				1,104	1,435	1,766	2,097	2,429	2,760		13
34				1,255	1,632	2,008	2,385	27,62	3,138		16
36				1,416	1,841	2,266	2,690	3,115	3,540		8
38				1,586	2,062	2,538	3,013	3,489	3,964		4
40				1,766	2,295	2,824	3,354	3,883	4,412		3
42					2,540	3,126	3,712	4,297	4,883	5,469	2
44					2,797	3,442	4,087	4,732	5,377	6,022	3
46					3,067	3,773	4,480	5,187	5,894	6,601	3
48					3,348	4,119	4,891	5,662	6,434	7,205	7
50					3,641	4,480	5,319	6,158	6,997	7,836	2
52					3,946	4,855	5,764	6,673	7,583	8,492	2
54					4,263	5,246	6,228	7,210	8,192	9,174	3
56					4,593	5,650	6,708	7,766	8,824	9,881	0
58					4,934	6,070	7,206	8,342	9,478	10,615	1
60					5,287	6,505	7,722	8,939	10,156	11,374	0

1/ Based on weighted equation:

$$V = -0.22256D^2 + 0.34399D^2H - 21.06369H + \frac{7219.07130}{D^2} .$$

Standard error of estimate = 118.03 board feet or 8.57 percent of the mean volume.

2/ 12-inch class includes trees 11.0 to 12.9 inches d.b.h.

3/ Computed for 16.3-foot log lengths between logger stump and a top equaling 40 percent of d.b.h., but never less than 6.0 inches.

4/ Number of trees; range of data for 266 trees enclosed by solid line.

Table 4.--*Volume table for old-growth Sitka spruce in southeast Alaska*[1/]

(In board-feet, International 1/4-inch rule)

D.b.h. (D)[2/] (inches)	Merchantable height in 16-foot logs (H)[3/]										Basis[4/]
	1	2	3	4	5	6	7	8	9	10	
12	54	92	129	166	204	241					5
14		104	160	216	272	327					15
16		125	202	279	356	433					22
18		152	253	354	456	557					24
20		185	313	441	569	697					26
22			380	538	696	854	1,012	1,170			28
24			455	646	836	1,027	1,217	1,408			19
26			537	763	989	1,215	1,441	1,667			18
28			627	891	1,155	1,419	1,683	1,947			21
30			723	1,028	1,333	1,639	1,944	2,249			20
32				1,175	1,524	1,874	2,223	2,572	2,921		13
34				1,332	1,728	2,124	2,520	2,916	3,312		16
36				1,499	1,944	2,390	2,835	3,281	3,726		8
38				1,675	2,173	2,671	3,168	3,666	4,164		4
40				1,861	2,414	2,967	3,520	4,073	4,626		3
42					2,667	3,278	3,890	4,501	5,112	5,723	2
44					2,933	3,605	4,277	4,950	5,622	6,294	3
46					3,211	3,947	4,683	5,419	6,155	6,891	3
48					3,502	4,305	5,107	5,910	6,712	7,515	7
50					3,805	4,677	5,549	6,421	7,293	8,165	2
52					4,121	5,065	6,009	6,953	7,898	8,842	2
54					4,449	5,468	6,487	7,507	8,526	9,545	3
56					4,789	5,887	6,984	8,081	9,178	10,275	0
58					5,142	6,320	7,498	8,676	9,854	11,032	1
60					5,507	6,769	8,030	9,292	10,553	11,815	0

1/ Based on weighted equation:

$$V = -0.22276D^2 + 0.35417D^2H - 13.51970H + \frac{7011.99610}{D^2}.$$

Standard error of estimate = 122.56 board feet or 8.90 percent of the mean volume.

2/ 12-inch class includes trees 11.0 to 12.9 inches d.b.h.

3/ Computed for 16.3-foot log lengths between logger stump and a top equaling 40 percent of d.b.h., but never less than 6.0 inches.

4/ Number of trees; range of data for 266 trees enclosed by solid line.

Table 5.--*Volume table for old-growth Sitka spruce in southeast Alaska*[1/]

(In cubic feet, Smalian formula)

D.b.h. (D)[2/] (inches)	Merchantable height in 16-foot logs (H)[3/]										Basis[4/]
	1	2	3	4	5	6	7	8	9	10	
12	20	24	28	33	37	41					5
14		30	36	43	50	56					15
16		36	45	55	65	74					22
18		42	55	68	81	95					24
20		50	67	84	100	117					26
22			79	100	121	142	163	184			28
24			93	119	144	170	195	221			19
26			108	139	169	200	230	261			18
28			124	160	196	232	268	304			21
30			142	183	225	267	309	350			20
32				208	256	304	352	400	448		13
34				234	389	343	398	452	507		16
36				262	324	385	447	508	570		8
38				292	361	429	498	567	636		4
40				323	399	476	553	629	706		3
42					440	525	610	695	779	864	2
44					483	577	670	763	857	950	3
46					528	630	733	835	937	1,040	3
48					575	687	798	910	1,022	1,133	7
50					624	745	867	988	1,100	1,231	2
52					674	806	938	1,069	1,201	1,333	2
54					727	870	1,012	1,154	1,296	1,438	3
56					782	935	1,088	1,242	1,395	1,548	0
58					839	1,003	1,168	1,333	1,497	1,662	1
60					898	1,074	1,250	1,427	1,603	1,779	0

1/ Based on weighted equation: $V = 16.30566 - 3.09606H + 0.04983D^2H$. Standard error of estimate = 22.02 cubic feet or 9.80 percent of the mean volume.

2/ 12-inch class includes trees 11.0 to 12.9 inches d.b.h.

3/ Computed for sections under 20 feet in length between logger stump and 4-inch d.i.b. top.

4/ Number of trees; range of data for 266 trees enclosed by solid line.

Table 6.--*Volume table for old-growth western hemlock in southeast Alaska*[1/]

(In board feet, Scribner rule)

D.b.h. (D)[2/] (inches)	Merchantable height in 16-foot logs (H)[3/]										Basis[4/]
	1	2	3	4	5	6	7	8	9	10	
12	32	64	97	129	161						18
14	47	94	141	188	235	282	328				44
16		128	192	256	320	384	448				84
18		166	250	333	416	499	582				70
20		209	314	419	524	629	733				65
22		257	386	514	643	771	900				69
24			464	619	773	928	1,083	1,237			62
26			549	732	915	1,098	1,281	1,464			75
28			641	854	1,068	1,282	1,495	1,709			35
30			739	986	1,232	1,479	1,725	1,972			39
32			845	1,127	1,408	1,690	1,972	2,253			33
34			957	1,276	1,595	1,914	2,233	2,553			30
36			1,076	1,435	1,794	2,153	2,511	2,870			34
38			1,202	1,603	2,004	2,404	2,805	3,206			22
40			1,335	1,780	2,225	2,670	3,115	3,559			14
42				1,966	2,457	2,949	3,440	3,931	4,423		17
44				2,161	2,701	3,241	3,781	4,321	4,862		13
46				2,365	2,956	3,547	4,138	4,730	5,321		9
48				2,578	3,222	3,867	4,511	5,156	5,801		5
50				2,800	3,500	4,200	4,900	5,601	6,301		4
52					3,789	4,547	5,305	6,063	6,821	7,579	3
54					4,090	4,908	5,726	6,544	7,362	8,180	0
56					4,402	5,282	6,162	7,043	7,923	8,804	2
58					4,725	5,670	6,615	7,560	8,505	9,450	0
60					5,059	6,071	7,083	8,095	9,107	10,119	0

1/ Based on weighted equation: $V = -8.63080H + 0.28348D^2H$. Standard error of estimate = 115.22 board feet or 11.12 percent of the mean volume.

2/ 12-inch class includes trees 11.0 to 12.9 inches d.b.h.

3/ Computed for 16.3-foot log lengths between logger stump and a top equaling 40 percent of d.b.h., but never less than 6 inches.

4/ Number of trees; range of data for 747 trees enclosed by solid line.

Table 7.--*Volume table for old-growth western hemlock in southeast Alaska*[1/]

(In board feet, International 1/4-inch rule)

D.b.h. (D)[2/] (inches)	Merchantable height in 16-foot logs (H)[3/]										Basis[4/]
	1	2	3	4	5	6	7	8	9	10	
12	48	87	126	165	203						18
14	57	113	168	224	279	335	390				44
16		144	218	293	368	443	517				84
18		179	275	372	469	565	662				70
20		218	339	460	581	702	823				65
22		262	409	557	705	853	1,001				69
24			486	664	841	1,019	1,196	1,374			62
26			569	779	989	1,198	1,408	1,617			75
28			659	903	1,147	1,392	1,636	1,880			35
30			755	1,036	1,318	1,599	1,881	2,162			39
32			857	1,178	1,500	1,821	2,142	2,463			33
34			965	1,329	1,693	2,056	2,420	2,784			30
36			1,080	1,489	1,897	2,306	2,715	3,123			34
38			1,201	1,657	2,114	2,570	3,026	3,482			22
40			1,329	1,835	2,341	2,847	3,354	3,860			14
42				2,021	2,580	3,139	3,698	4,256	4,815		17
44				2,216	2,830	3,444	4,058	4,672	5,286		13
46				2,420	3,092	3,764	4,436	5,107	5,779		9
48				2,633	3,365	4,097	4,830	5,562	6,294		5
50				2,855	3,650	4,445	5,240	6,035	6,830		4
52					3,946	4,806	5,667	6,527	7,388	8,248	3
54					4,253	5,182	6,111	7,039	7,968	8,896	0
56					4,572	5,571	6,571	7,570	8,569	9,568	2
58					4,902	5,975	7,047	8,120	9,192	10,264	0
60					5,244	6,392	7,540	8,688	9,836	10,985	0

1/ Based on weighted equation: $V = -9.6512 - 7.4021H + 2.6511D - 0.1795\,D^2 + 0.3210D^2H + \frac{1876.4131}{D^2}$

Standard error of estimate = 119.55 board feet or 10.78 percent of the mean volume.

2/ 12-inch class includes trees 11.0 to 12.9 inches d.b.h.

3/ Computed for 16.3-foot log lengths between logger stump and a top equaling 40 percent of d.b.h., but never less than 6 inches.

4/ Number of trees; range of data for 747 trees enclosed by solid line.

Table 8.--*Volume table for old-growth western hemlock in southeast Alaska*[1/]

(In cubic feet, Smalian formula)

D.b.h. (D)[2/] (inches)	Merchantable height in 16-foot logs (H)[3/]										Basis[4/]
	1	2	3	4	5	6	7	8	9	10	
12	15	20	26	31	37						18
14	19	26	34	42	49	57	65				44
16		34	44	54	64	74	84				84
18		42	55	67	80	93	105				70
20		52	67	83	99	114	130				65
22		62	81	100	119	138	157				69
24			96	119	141	164	186	209			62
26			113	139	166	192	219	245			75
28			131	162	192	223	254	284			35
30			150	185	221	256	291	326			39
32			171	211	251	291	331	371			33
34			193	238	283	328	374	419			30
36			216	267	317	368	419	469			34
38			241	297	354	410	467	523			22
40			267	329	392	454	517	579			14
42				363	432	501	570	639	708		17
44				398	474	550	625	701	777		13
46				435	518	601	684	766	849		9
48				474	564	654	744	834	925		5
50				514	612	710	808	905	1,003		4
52					662	768	873	979	1,085	1,191	3
54					714	828	942	1,056	1,170	1,284	0
56					768	890	1,013	1,136	1,258	1,381	2
58					823	955	1,087	1,218	1,350	1,481	0
60					881	1,022	1,163	1,304	1,444	1,585	0

1/ Based on weighted equation: $V = 0.04915D^2 + 0.03912D^2H + \frac{269.92685}{D^2}$. Standard error of estimate = 19.11 cubic feet or 10.49 percent of the mean volume.

2/ 12-inch class includes trees 11.0 to 12.9 inches d.b.h.

3/ Computed for sections under 20 feet in length between logger stump and 4-inch d.i.b. top.

4/ Number of trees; range of data for 747 trees enclosed by solid line.